1962

Transpiration cooling of a heat generating porous plate

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TRANSPIRATION COOLING OF A HEAT GENERATING POROUS PLATE

BY

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A

THESIS

submitted to the faculty of the

SCHOOL OF MINES AND METALLURGY OF THE UNIVERSITY OF MISSOURI

in partial fulfillment of the work required for the

Degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

Rolla, Missouri

1962

Approved by

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ABSTRACT

This investigation is a continuation of a thesis by T. S. Purewal (7). Several of the simplifying assumptions made by the above author are either ignored or the justifiability of such assumptions is probed into in order to obtain a more rigorous solution. Effects of the assorted grain size, non-Newtonian cooling, direction of coolant flow, thickness of the porous heat generating plate, specific heat of the coolant, and compressibility of the coolant are investigated in this thesis. Variation of maximum temperature of both coolant and the solid material for different types of heat source distribution are also studied.

As the grain size decreases heat transfer area per unit volume increases considerably. It is also observed that the temperature difference between the solid and the coolant increases as the particle size increases. It is established that the assumption of Newtonian cooling does not introduce any error in the case of a porous plate formed by very small spheres. Expressions are also derived for maximum coolant and solid temperatures under uniform, linear, exponential and sinusoidal heat source distribution. The effect of thickness of the plate on heat distribution and on the temperature of the solid and the coolant have been mathematically expressed. The specific heat of coolant fluid does affect the temperature of the coolant. Specific heats of different fluids are not equal. Even for the same fluid specific heat generally varies with temperature. How these variations affect the coolant temperature was studied and equations for different categories of heat source distribution developed. Graphs based on such

equations are presented. In the case of compressible fluids higher pressures are found to be advantageous. Advantages include lower percentage pressure drop, lower figure of merit and higher mass rate of flow for same linear velocity.

ACKNOWLEDGEMENTS

In undertaking an investigation such as the one which culminated in this thesis the investigator has to depend more than anything else on his advisor. Depth of knowledge, imagination, insight, interest, eagerness, understanding and willingness to help, are all qualities which the former anxiously craves the latter to possess. In a work that is to be started practically from scratch, the above factors assume more significance than ever. The author found himself in such a position when he started working on this investigation. But he has been singularly lucky to have an advisor possessing all the aforementioned qualities in copious abundance. It is with heartfelt feelings of most sincere appreciation that the author places on record his indebtedness to his advisor, Dr. Aaron J. Miles.

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I. INTRODUCTION

Progress brought about by human ingenuity and technological developments has always been accompanied by fresh problems the successful solution of which enables still further progress. These problems may be simple or complex. Many of them may elude solution for a period that is considerable. The history of technological development is nothing but the story of man's triumphant struggle against such problems. The nuclear power development opened a new era. The recent developments in space explorations and exploitations are astounding. The coming years are likely to bring forth many more remarkable achievements. But for several engineering problems encountered side by side with these developments the pace of progress would surely have been much more rapid. One of such problems is the removal of heat generated during a process or operation. Anxious are the moments when a manned space capsule reenters the earth's atomsphere after orbital flight. What causes most concern is probably the thought of whether it would be able to withstand the heat generated owing to friction. Different techniques have been developed and employed to meet such situations. Transpiration cooling is not yet one. It may not be very long before this young branch of heat transfer technology is thoroughly developed and fully employed. It may provide the key to many of the vexing heat transfer problems of today.

The principle of transpiration cooling is simple. A coolant fluid (liquid or gas) is allowed to pass through the pores of a porous body that is to be cooled. The coolant comes into contact all around the particles forming the heated body and removes heat. Very large heat transfer area per unit volume and intimate contact between the heated body and the coolant are the obvious advantages of transpiration cooling. It is an elementary heat transfer principle that the rate of heat removal is directly proportional to the heat transfer area, the overall heat transfer coefficient and the temperature gradient. An increase in any one of these will result in a corresponding increase in heat transmission rate. So also, for the same quantity of heat to be removed, an increase in one factor will permit a corresponding decrease in any one or both of the remaining factors. For example, if the heat transfer area is doubled and if the heat transfer coefficient remains unchanged, the temperature gradient could be reduced to one half of the original to maintain the same rate of heat removal.

Transpiration cooling can be applied where internal heat generation is occurring or not. This field has already engaged the attention of a few workers. The extent of work done is not negligible; but it cannot be claimed to be complete. Investigations regarding the effectiveness of transpiration cooling in systems with internal heat generation however are ~~is~~ meagre. Investigation by T. S. Purewal (7) done in this field was preceded only by the work of L. Green, Jr. (3) as far as publications reveal. These were preliminary investigations and very much limited by many simplifying assumptions. Some of the assumptions might be justifiable while others might be sufficient to undermine the accuracy and reliability of the calculations. It is up to the further investigators to tackle the problem in more rigorous manner taking into consideration actual conditions and to develop and perfect the technology of transpiration cooling as an infalliable and reliable weapon.

It might be possible to solve an engineering problem in a variety of ways using a variety of techniques, just as it might be possible to cure any particular disease by more than one medicine or by more than one type of treatment. To choose the method that is most efficient and least expensive is the duty of the engineer. Anything that makes the technique less expensive without diminishing the efficiency can be termed an improvement. In that sense, in the field of high temperature heat transfer such as in the nuclear reactors, space craft, high temperature furnaces etc., the transpiration cooling system is bound to win recognition as an improvement. But before it can win that honor the process has to be thoroughly probed into and developed into a reliable piece of technology.

II. TABLE OF UNITS AND SYMBOLS

Symbol	Description	Units
A	Cross sectional area of porous solid	Ft.^{2}
C	Sp. heat of coolant	Btu/lb.°F.
d	Diameter of the grains	Ft.
f	Porosity	
G	Mass rate of flow of the coolant	Lb/Ft.^{2}hr.
h	Heat transfer coefficient	Btu/hr.°F.Ft.^{2}
l	Thickness of the plate	Ft.
K	Thermal conductivity	Btu/hr.°F.Ft.^{2}/Ft.
q	Volumetric heat generation per unit volume	Btu/hr.Ft.^{3}
t	Temperature of the coolant	°F
T	Temperature of the solid	°F
$\Delta\theta$	Temperature difference between solid and coolant	°F
r	Radius of heat generating spherical particles	Ft.
T_i	Temperature of the center of the sphere	°F
T_s	Temperature at the surface of the sphere	°F

III. REVIEW OF LITERATURE

Transpiration cooling as a heat transfer technique has been in the active consideration of scientists and engineers only for the past two decades or so. During this short span various aspects of the process in general have been investigated by several research workers. However, cases where internal heat generation is involved have not been probed into by many.

W. D. Rannie (1) appears to be one of the earliest scientists who investigated porous wall cooling. The jet propulsion lab: publication of the California Institute of Technology published in 1947, contains details of Rannie's theoretical studies of heat transfer in a sweat cooled duct. In the same year the U. S. Navy project squid technical report No. 4 (2) brought to light details of S. W. Yuan's theoretical investigation of temperature field in laminar boundary layer on a porous flat plate with fluid injection. H. L. Wheeler (20) developed experimental techniques in investigating the influence of different variables in porous plate cooling.

The credit for pioneering work on transpiration cooling of porous heat generating source goes to Leon Green, Jr. (3). His paper under the caption 'Gas Cooling of a Porous Heat Source' was published in the Journal of Applied Mechanics. This was in 1952. He examined the solid to fluid heat transfer in which a gas passes through a porous wall of high specific surface with heat generation within the solid material. He assumed that in the porous wall gas temperature and solid temperature are approximately equal. It was also assumed that the thermal conductivity of the solid and the specific heat of the gas are constants. An equation developed by Green for the calculation of pressure drop across the wall is found to be very useful. Green did not limit his investigations to purely heat transfer aspects alone. From the consideration of the specific case of a Helium cooled graphite wall he concluded that efficient operation from the point of view of pumping power would require the use of a system under a pressure of several atomspheres. Green's equation for calculating the energy required to pump the coolant will be found elsewhere in this thesis. His graphs correlating pressure drop to flow rate, pumping power/power output ratio to flow rate for different inlet pressures and minimum pumping power/power output ratio to average pressure, are appended to this thesis.

The Mechanical Engineering Department of the Missouri School of Mines and Metallurgy has done and continues to do much work in this field. In a short period of two years four theses have been produced. In 1961 Herbert S. Brahinsky (4) investigated transpiration cooling of a porous plate with coolant flowing counter to heat flow. He reported that plate temperature depends upon heat flux, coolant flux, thermal conductivity of the solid material, specific heats of the solid and the fluid, density of the solid and the porosity. He also reported that in the steady state the maximum temperature at the surface is dependent only on the heat flux and the mass rate of flow of the coolant. Transient temperatures in a porous plate for one dimensional counter coolant flow were determined by R. G. Posgay (5). He considered a porous plate one surface of which was heated by a hot gas. A thesis submitted by Wm. C. Wolkenhauer (6) is similar to the above. In the year 1962, T. S. Purewal (7) reported results of his investigations on the transpiration cooling of a heat generating porous plate. His study was limited to Newtonian cooling of a heat generating porous plate composed of uniform spherical particles cooled by an incompressible coolant whose specific heat remained unaffected during the process. His assumption that at any point in the plate, temperature of the solid and that of the coolant are not necessarily the same is a major deviation from previous investigators. His study covered uniform, linear, exponential and sinusoidal distributions of the internal heat generated. Expressions for coolant and solid temperatures at any point in the plate were developed and presented by him. Expressions were established also for temperature difference between the heat generating plate and the coolant.

Chapter Nine of 'Conduction Heat Transfer' by P. J. Schneider (8) deals with transpiration cooling. This chapter includes a short discussion of transpiration cooling where internal heat generation is involved.

Many of the publications reviewed are not concerned with heat transfer in particular. They have, however, been very useful in understanding many factors that affect porous plate transpiration cooling indirectly. The publication of Miles and Ponder (9) on porosity - permeability relationship throws much light on the effect of grain size and porosity on heat transfer surface area. Formulae enabling calculation of surface area per unit volume from knowledge of the particle shape and size, developed by them, have been used in this thesis.

Fancher and Lewis (10) are of the view that flow of fluids through porous media closely resembles that through pipes. They claim that there is a condition of flow in porous systems which resembles laminar flow and another which corresponds to turbulent.

Chalmers, Joseph, Taliaferro and Hawlins (11) produced a plot of modified Reynold's number against Fanning's friction factor for flow of fluids through porous media. They replaced d representing diameter in the Reynold's number by d_e and named it as equivalent diameter. The 'equivalent diameter' was defined as mean effective pore diameter equivalent to the diameter of a capillary tube which would pass the same volume of fluid in the same time under equal pressure drop as would a single series of connected pores. Using the data for viscous flow only, this term was calculated from Poiseuille's law as follows:

$$\Delta P = \frac{32 \mu L U}{g \, d_e^2}.$$

$$d_e^2 = \frac{32 \mu L U}{g \, \Delta P}$$

μ = absolute viscosity - lb/sec. ft.

L = length of porous material core - ft.

g = acceleration due to gravity - 32.2 ft/sec^2

ΔP = pressure drop - lb/Ft^2

d_e = equivalent diameter - Ft.

Bartell (12) and co-workers used this term repeatedly and discussed it critically and regarded it as having more than hypothetical value. They hold that in the application of Poiseuille's law to flow through porous media it is essential to correct for (a) deviation of the cross section of the average pore from circular (b) increased length of path in such a system as compared to the apparent path (c) the larger pressure drop necessary for flow in a sinuous path (d) added energy consumption due to many alternate enlargements and contractions of the cross section in the average path of flow. C. S. Schlichter (13) has shown that in an assemblage of spheres of equal diameters the actual path of travel is from 1.2 to 1.5 times the apparent and that the average velocity is 1.8 times the apparent velocity. Chilton and Colburn (14) from theoretical reasoning claimed that in viscous flow through porous media, expansion and contraction account for at least 80% of the total resistance and friction due to viscosity of the fluid for 20% or less for a ratio of the minimum to maximum cross sectional area of a pore of 0.33. A correction factor on this basis when applied to the equation, $d_e^2 = \frac{32 \mu L U}{g \Delta P}$ will increase the value of the equivalent diameter.

A number of journals and books were perused by the author in connection with this investigation. They are acknowledged in the bibliography.

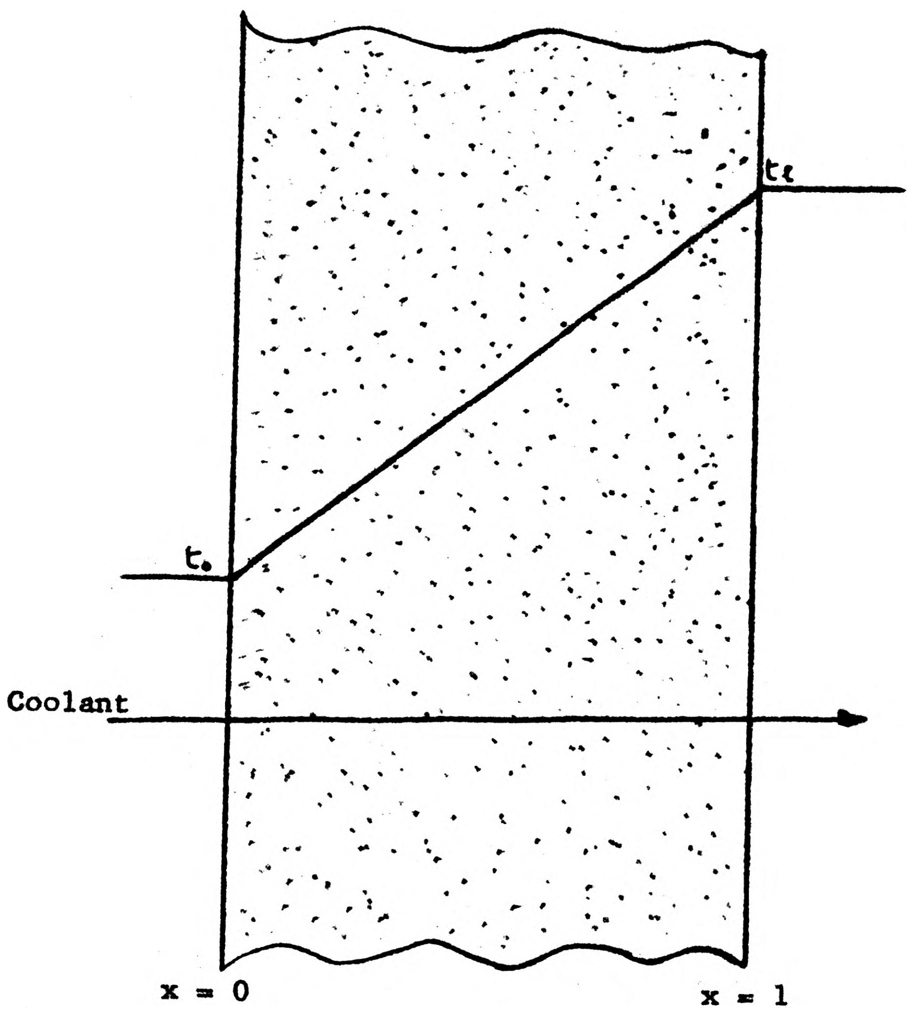


Fig. 1 Schematic Diagram

IV. DISCUSSION

The thesis by T. S. Purewal (7) of which this investigation is a continuation contained the following assumptions:

1. The thermal properties of the solid and the fluid do not change as a function of temperature.

2. The fluid flow and heat flow are steady and undirectional.

3. Cooling of the particles in the porous solid is Newtonian.

4. Temperature of the coolant in the pores and temperature of the adjacent solid are not identical.

5. The coolant fluid is incompressible and not heat generating.

In this investigation assumption No. 4 above is retained as such. All other assumptions are partly or wholly ignored or the justification is questioned. Wherever possible, expressions developed in the previous work are used. Symbols and units are maintained consistent, with a view to assure continuity. Figures assumed for the purpose of example problems and graphs are arbitrary. An effort has been made to make such figures as realistic as possible. The approach to this investigation is analytic.

EFFECT OF HEAT GENERATION FUNCTION ON THE MAXIMUM TEMPERATURE OF THE SOLID GRAINS OF THE POROUS MEDIA AND ON THE COOLANT.

a. Uniform Heat Generation.

$$t_x = \frac{q(1-f)x}{GC} + t_o \quad (7)$$

t_x becomes maximum when x = 1,

$$\therefore t_{max} = \frac{q(1-f)\ell}{GC} + t_o \qquad \text{. . . . .} \quad (1)$$

$$T_x = t_o + \frac{q(1-f)x}{GC} + \frac{qd}{6h} \quad (7)$$

$$T_{max} = T_\ell = t_o + \frac{q(1-f)\ell}{GC} + \frac{qd}{6h} \qquad \text{. . . . .} \quad (2)$$

b. Linear Heat Generation.

$$t_x = \frac{q_o(1-f)}{GC\ell}\left[\ell - \frac{x}{2}\right]x + t_o \quad (7)$$

When t is maximum,

$$\frac{dt}{dx} = 0$$

$$\frac{dt}{dx} = \frac{q_o(\ell-x)(1-f)}{GC\ell} = 0$$

$\therefore$ 1 = x when $t_x = t_{max}$.

$$\therefore t_{max} = \frac{q_o(1-f)\ell}{2GC} + t_o \qquad \text{. . . . .} \quad (3)$$

$$T_x = q_o\left[\frac{x(1-f)(2\ell-x)}{2GC\ell} + \frac{d(\ell-x)}{6h\ell}\right] + t_o \quad (7)$$

When $T_x = T_{max}$,

$$\frac{dT}{dx} = 0.$$

$$\therefore \frac{q_o}{\ell}\left[\frac{(1-f)(\ell-x)}{GC} - \frac{d}{6h}\right] = 0.$$

$\therefore$ When $T_x = T_{max}$,

$$x = 1 - \frac{GCd}{6h(1-f)} \quad \text{. . . . . (4)}$$

c. Sinusoidal Heat Generation.

$$t_x = t_o + \frac{q_o(1-f)\ell}{\pi GC}\left[1 - \cos\frac{\pi x}{\ell}\right] \quad (7)$$

When,

$$t_x = t_{max},$$

$$\frac{dt}{dx} = 0$$

$$\therefore \frac{q_o(1-f)}{GC} \sin\frac{\pi x}{\ell} = 0$$

$$\therefore \sin\frac{\pi x}{\ell} = 0$$

$$x = \ell.$$

$$\therefore t_{max} = t_o + \frac{2q_o(1-f)\ell}{\pi GC} \quad \text{. . . . . (5)}$$

$$T_x = t_o + \frac{q_o(1-f)\ell}{\pi GC}\left[1 - \cos\frac{\pi x}{\ell}\right] + \frac{q_o d}{6h} \cdot \sin\frac{\pi x}{\ell} \quad (7)$$

When $T_x = T_{max}$,

$$\frac{dT}{dx} = 0.$$

$$\therefore \frac{q_o(1-f)}{GC} \sin\frac{\pi x}{l} + \frac{q_o d\pi}{6hl} \cos\frac{\pi x}{l} = 0$$

$$\therefore \frac{(1-f)}{GC} \sin\frac{\pi x}{l} = -\frac{\pi d}{6hl} \cos\frac{\pi x}{l}.$$

$$\tan\frac{\pi x}{l} = -\frac{\pi d\, GC}{6hl(1-f)}$$

$\therefore$ When $T_x = T_{max}$,

$$x = \frac{l}{\pi} \tan^{-1}\left[-\frac{\pi d\, GC}{6hl(1-f)}\right] \qquad \text{. . . . . (6)}$$

d. Exponential Heat Generation.

$$t_x = t_o + \frac{q_o(1-f)}{KGC}\left[1 - e^{-Kx}\right] \quad (7)$$

When $t_x = t_{max}$,

$x = 1$

$$\therefore t_{max} = t_o + \frac{q_o(1-f)}{KGC}\left[1 - e^{-Kl}\right] \qquad \text{. . . . . (7)}$$

$$T_x = t_o + \frac{q_o(1-f)}{KGC} + q_o\left[\frac{d}{6h} - \frac{(1-f)}{KGC}\right] e^{-Kx} \quad (7)$$

When,

$$T_x = T_{max},$$

$$\frac{dT}{dx} = 0.$$

$$\frac{dT}{dx} = -q_0\left[\frac{d}{6h} - \frac{(1-f)}{KGC}\right]e^{-kx}.k.$$

$\therefore$ When,

$$T_x = T_{max},$$

$$\frac{1-f}{KGC} = \frac{d}{6h}.$$

Hence,

$$T_{max} = t_0 + \frac{q_0(1-f)}{KGC} \quad \ldots\ldots\ldots\ldots \quad (8)$$

EFFECT OF SPECIFIC HEAT OF THE COOLANT ON THE TEMPERATURE OF THE COOLANT.

Expressions for temperature of the coolant for different types of heat source distribution are shown below (7).

Type of heat distribution.	Corresponding Coolant Temperature (7)
Uniform heat generation	
$dQ = q(1-f)A\,dx$.	$t_x = t_0 + \frac{q(1-f)x}{GC}$.
Linear	
$q_x = q_0 \frac{(\ell - x)}{\ell}$.	$t_x = t_0 + \frac{q_0(1-f)}{GC\ell}\left[\ell - \frac{x}{2}\right] x$.
Exponential	
$q_x = q_0 e^{-Kx}$.	$t_x = t_0 + \frac{q_0(1-f)}{KGC}\left[1 - e^{-Kx}\right]$.
Sinusoidal	
$q_x = q_0 \sin\frac{\pi x}{\ell}$.	$t_x = t_0 + \frac{q_0(1-f)\ell}{\pi GC}\left[1 - \cos\frac{\pi x}{\ell}\right]$.
General	
$q_x = q_0 H(x)$.	$t_x = \frac{q_0(1-f)}{GC}\int H(x)\,dx$.

In all the above expressions c (sp. heat of the coolant) appears in the denominator. Therefore the temperature of the fluid is inversely proportional to its specific heat. If a fluid of high specific heat is used as the coolant more heat can be removed for the same temperature rise relative to a low specific heat coolant. Specific heat of different materials differ. Even for the same material specific heat generally changes with temperature. Specific heats of some of the fluids which may be useful as coolants in transpiration cooling are tabulated and shown in the appendix.

Variations of maximum temperature of the coolant with specific heat are plotted for the different types of internal heat generation and presented in Fig. 2, 3, 4 and 5.

DATA FOR FIGURES 2, 3, 4 AND 5

$q = 2 \times 10^7$ Btu/ft^{3}hr.

$1 = 0.25'$

$K = 5$

$C = 0.2$ to 2

$G = 2.36 \times 10^5$ lb/hr.ft^2

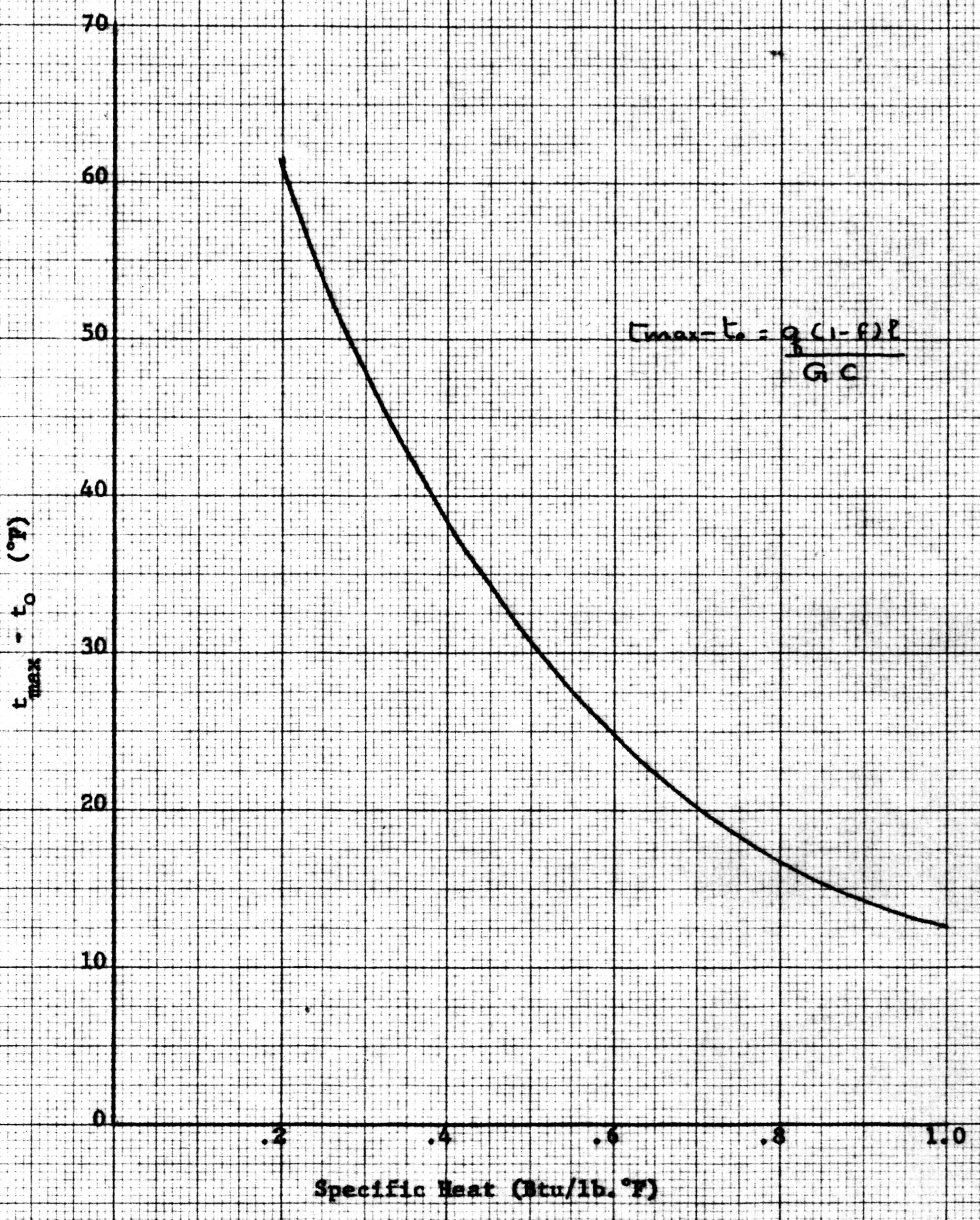


Fig. 2 Effect of Specific Heat on Maximum Coolant Temperature - Uniform Heat Generation

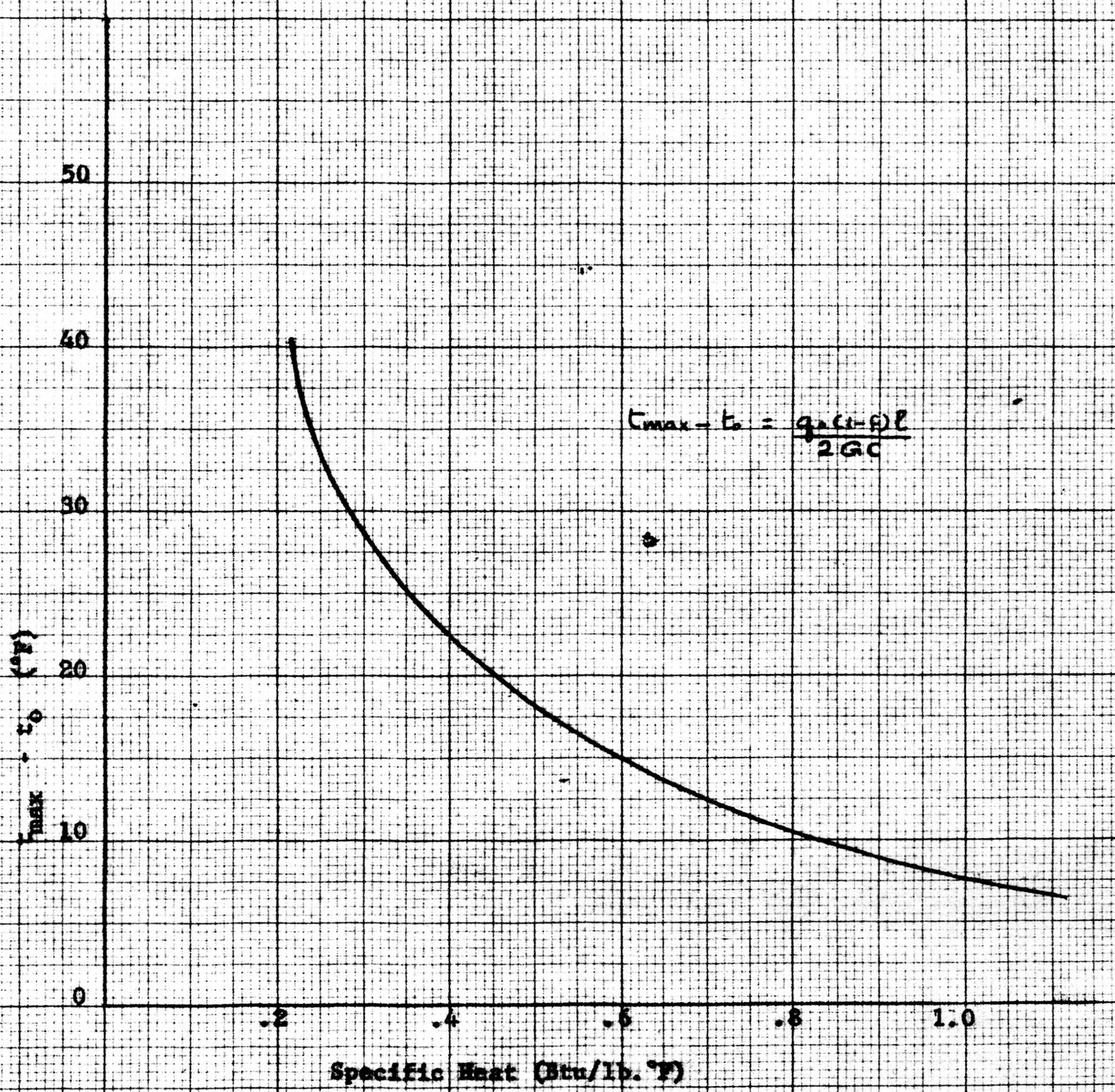


Fig. 3 Effect of Specific Heat on Maximum Coolant Temperature - Linear Distribution of Heat

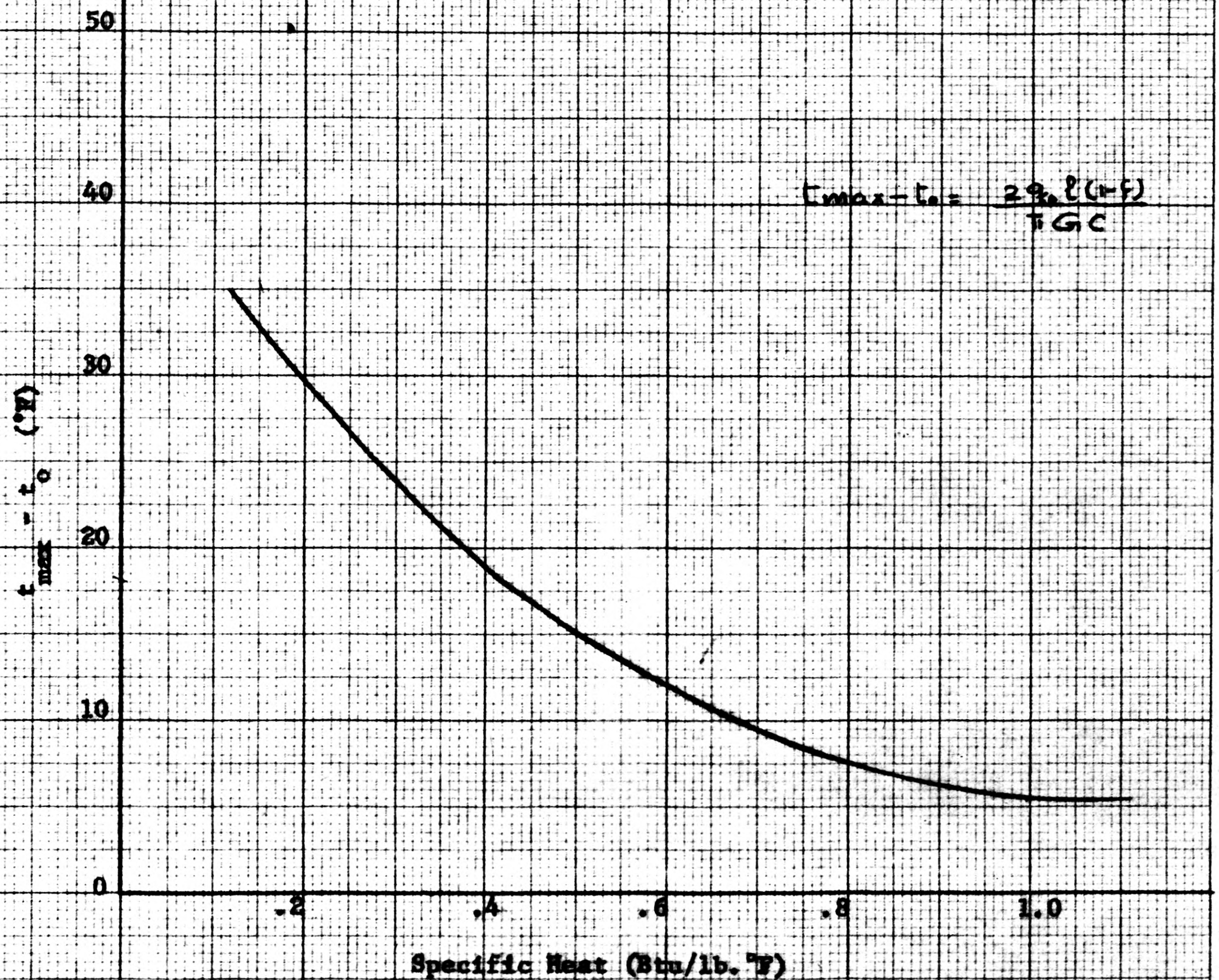


Fig. 4 Effect of Specific Heat on Maximum Coolant Temperature - Sine Distribution of Heat.

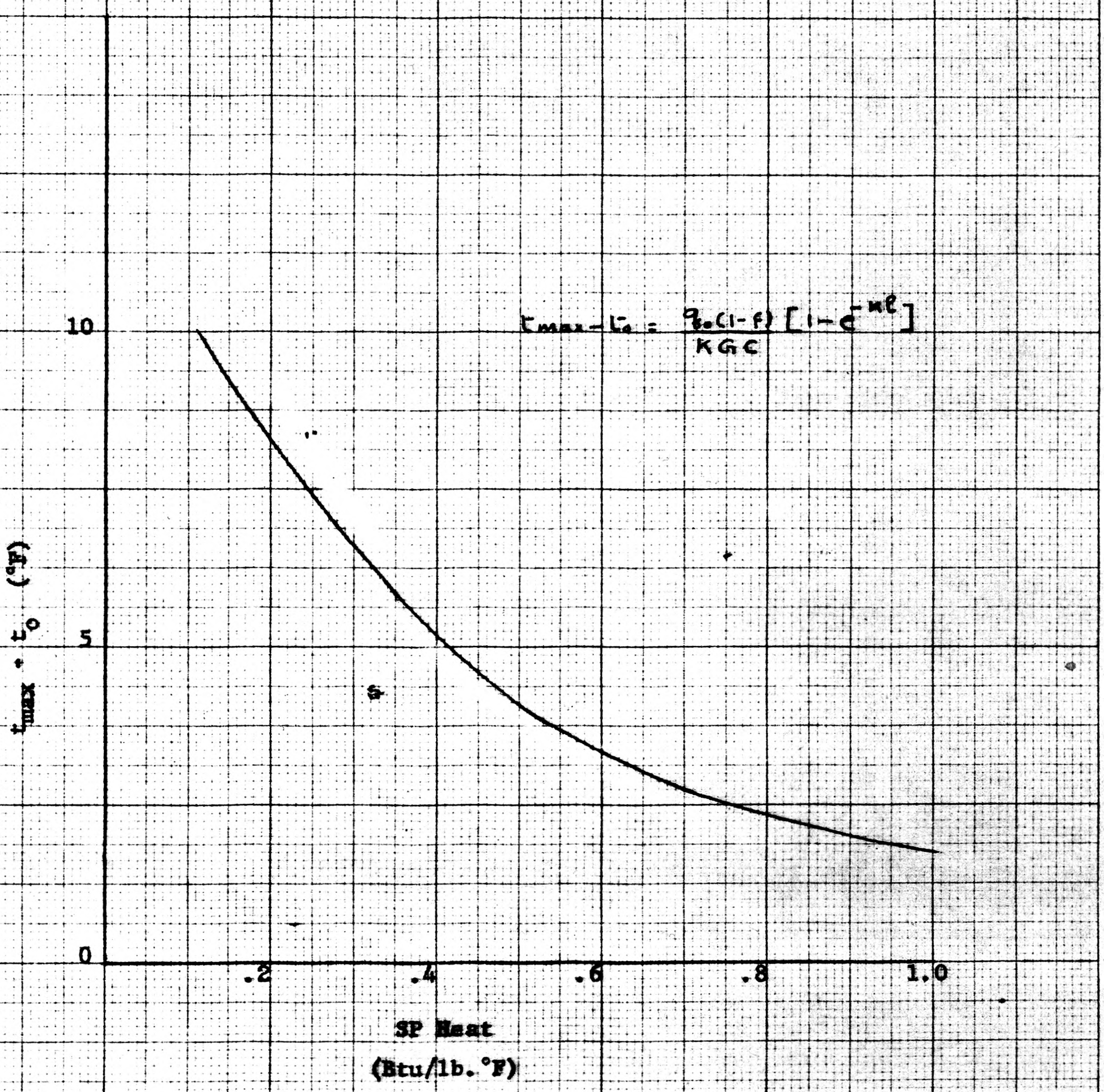


Fig. 5 Effect of Specific Heat on Maximum Coolant Temperature - Exponential Distribution of Heat.

EFFECT OF THE THICKNESS OF THE PLATE.

An inspection of the equations for different types of heat distribution is made below to determine the effect of the thickness of the generating porous plate on the rate of heat distribution and on the maximum temperature of the coolant.

a. Uniform Heat Generation.

$$dQ = q\,(1-f)\,A\,dx$$

In this case the thickness of the plate has no effect on the rate of heat generation.

$$t_{max} = \frac{q\,(1-f)\,\ell}{GC} + t_o$$

As the thickness of the plate increases maximum temperature of the coolant increases.

b. Linear Heat Generation.

Volumetric heat generation is given by the following expression:

$$q_x = q_o\frac{(\ell - x)}{\ell}$$

This equation can be transformed as,

$$q_x = q_o - q_o\frac{x}{\ell} \qquad \text{. . . . . (8)}$$

Therefore the rate of volumetric heat generation increases as the thickness of the plate increases according to the above formula.

Also,

$$t_x = \frac{q_o\,(1-f)}{GC\ell}\left[\ell - \frac{x}{2}\right]x + t_o$$

rewriting,

$$t_x = t_o + \frac{q_o\,(1-f)\,x}{GC} - \frac{q_o\,(1-f)\,x^2}{2GC\ell} \qquad \text{. . . . . (9)}$$

As the thickness of the plate increases coolant temperature increases according to the above equation.

c. Sinusoidal Heat Generation.

$$q_x = q_o \sin\frac{\pi x}{l}.$$

When x = 0,

$$\sin\frac{\pi x}{l} = 0.$$

When x = 1,

$$\sin\frac{\pi x}{l} = 0.$$

When x = 1/2, heat generated will be maximum.

$$t_x = t_o + \frac{q_o(1-f)l}{\pi G C}\left[1 - \cos\frac{\pi x}{l}\right]$$

$$\therefore t_x = t_o + \frac{q_o(1-f)l}{\pi G C} - \frac{q_o(1-f)l}{\pi G C}\cos\frac{\pi x}{l} \quad \ldots\ldots\ldots\ldots (10)$$

When the value of l is such that $\cos\frac{\pi x}{l}$ is unity,

$$t_x = t_o$$

This can happen only when $\frac{x}{1}$ is nearly zero. In other words 1 must be very large in comparison with x.

$\cos\frac{\pi x}{l} = -1$, when $\cos\frac{\pi x}{l} = \cos\pi$.

When 1 is such that $\frac{x}{1}$ is nearly equal to unity,

$$t_x = t_o + \frac{2 q_o(1-f)l}{\pi G C}.$$

When 1 is such that $\frac{1}{x} = 2$, $\cos\frac{\pi x}{l} = 0$.

Then,

$$t_x = t_o + \frac{q_o(1-f)l}{\pi G C}.$$

EFFECT OF COMPRESSIBILITY OF THE COOLANT.

In transpiration cooling compressible as well as incompressible fluids can be used as coolants. Certain applications for which transpiration cooling is particularly adaptable are prone to find gases more convenient than liquids. Rocket cooling is an example. Nuclear reactors prefer Helium to other coolants. Gases under pressure aid heat transfer by virtue of increased mass rate of flow for the same volume rate of flow. A relatively smaller percentage of pressure drop is another advantage. These statements are substantiated in the following steps.

$$dq = A\,G\,C\,dt.$$

$$G = \rho V \quad .$$

The density of gases increase in direct proportion to pressure. By increasing the pressure to ten times, the heat transfer rate can be stepped up by ten times for the same volumetric rate of flow.

Pressure drop in porous bodies can be calculated from the following formula: (10)

$$\Delta P = \frac{f\,l\,\rho\,V^2}{d_e\,2\,g}.$$

Pressure drop is directly proportional to the density of the fluid and to the square of the linear velocity. By increasing the pressure, the same mass rate of flow can be maintained at a lower linear velocity. This will be helpful in reducing the pressure drop.

High pressure and resulting reduced percentage pressure drop are bound to ensure better economy as far as the pumping of the coolant fluid is concerned. Leon Green, Jr. (3) has shown that 'figure of merit' diminishes considerably with increasing pressure. Figure of merit as commonly applied to heat exchangers is defined as the ratio of the power required for pumping the fluid to the power removed as thermal energy.

Power required to pump a gas through the porous wall may be represented by the formula,

$$W = G \int_{P_1}^{P_2} \frac{dP}{\rho} .$$

For a perfect gas

$$\rho = \frac{P}{RT}$$

$$W = RTG \ln \frac{P_1}{P_2} .$$

Rate of heat removal is,

$$Q = G C \Delta T$$

Figure of Merit,

$$\frac{W}{Q} = \frac{RT}{C \Delta T} \ln \frac{P_1}{P_2}$$

For small pressure drops relative to the average absolute pressure,

$$\frac{W}{Q} = \frac{RT}{C \Delta T} \cdot \frac{P_1 - P_2}{P_{av}}$$

Green has suggested the use of average temperature in place of T. Higher average pressure tends to reduce the W/Q ratio.

EFFECT OF THE GRAIN SIZE.

Ponder and Miles (9) have developed mathematical relations expressing surface area per unit volume of porous bodies composed of grains of different geometrical shapes. A table containing such formulae will be seen in the appendix. For spherical particles the expression is,

$$\frac{S}{T} = \frac{6}{d}.$$

S represents average surface area, T the average volume and d the diameter of the spherical grain.

It is obvious from this expression that as the particle size decreases, the surface area per unit volume increases. This is important in heat transfer, as one of the vital factors affecting rate of heat transfer is the surface over which the process is taking place.

T. S. Purewal has shown that as the particle size decreases, temperature difference between the solid and the coolant decreases. In this respect, decrease in particle size does not augur well as far as heat transfer rate is concerned. The conflicting influence of the variation of particle size from the above two angles poses a problem. For obtaining the most favorable condition, an optimum particle size will have to be chosen.

As the grain size decreases the pore diameter becomes smaller too. This will cause increased pressure drop due to friction as the coolant flows through the pores.

$$h_L = f \frac{l V^2}{d_e 2g}$$

d_e represents the pore diameter
h_L represents head lost due to friction

If the porous plate is composed of assorted sizes, it might give rise to some complications. Primarily the effect of such assortment is to reduce average grain size. How assortments of different grain sizes affect surface area per unit volume can be seen from table I.

A maleffect of the assortment is that it might give rise to non-uniform pore diameter.

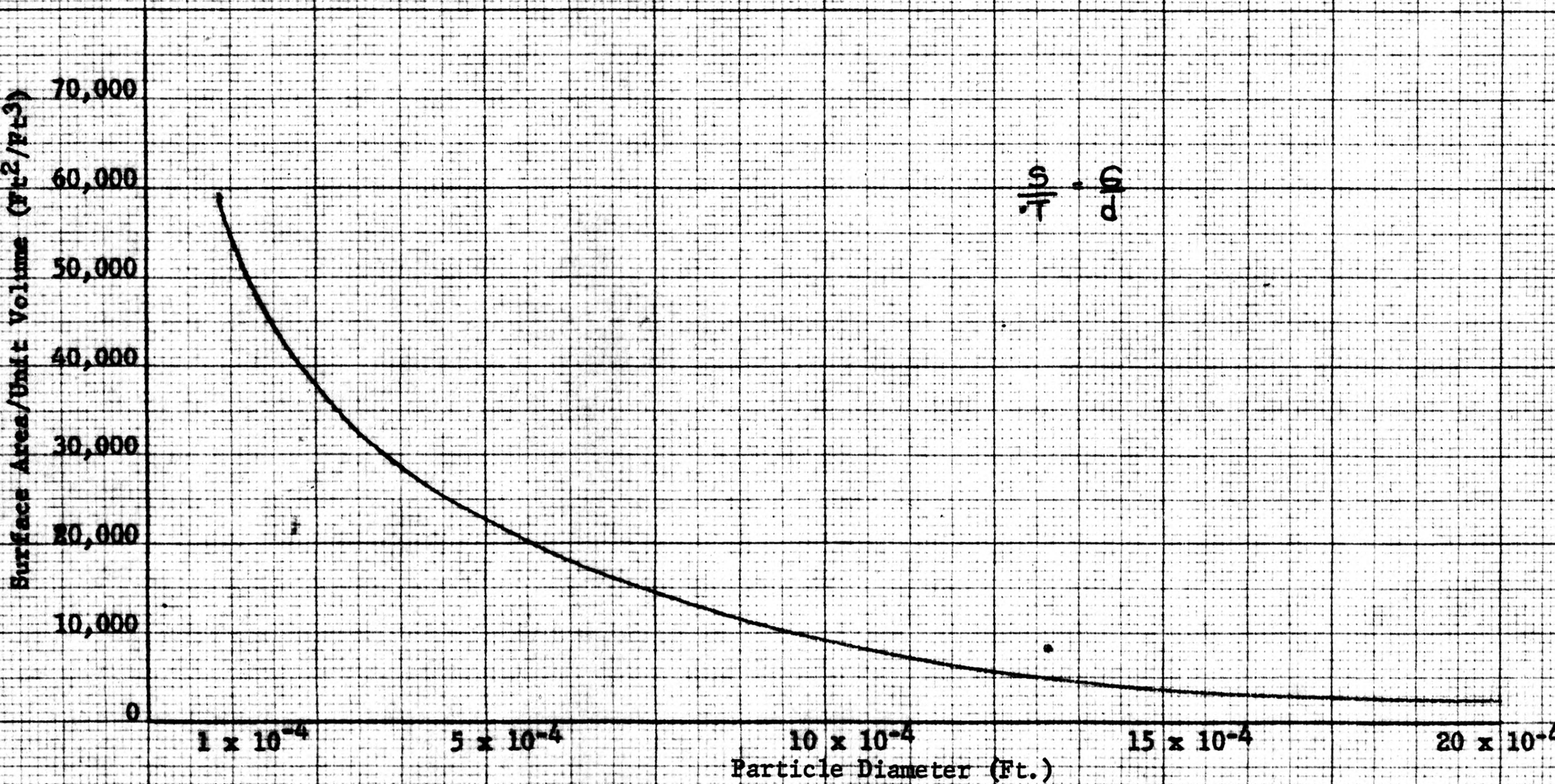


Fig. 6 Particle Diameter (Ft.) Vs Surface Area/Unit Volume (Ft^2/Ft^3)

TABLE I

EFFECT OF ASSORTED GRAIN SIZE ON SURFACE AREA PER UNIT VOLUME

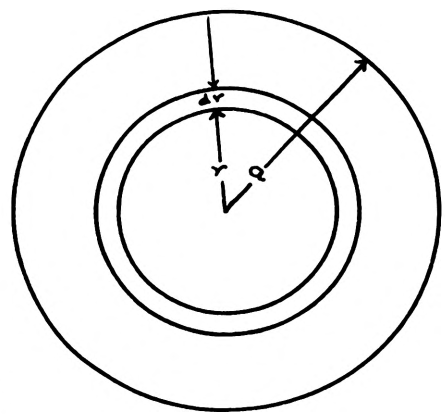
Particle Diameter (Ft.)	Percentage Per Unit Volume	Surface Area Per Unit Volume 1/Ft.	
10×10^{-4}	100	6×10^{3}	
20×10^{-4}	100	3×10^{3}	
10×10^{-4}	50	3×10^{3}	4.5×10^{3}
20×10^{-4}	50	1.5×10^{3}	
10×10^{-4}	25	1.5×10^{3}	4.37×10^{3}
13.33×10^{-4}	25	1.22×10^{3}	
16.67×10^{-4}	25	0.9×10^{3}	
20×10^{-4}	25	0.75×10^{3}	
10×10^{-4}	20	1.2×10^{3}	4.25×10^{3}
12.5×10^{-4}	20	0.96×10^{3}	
15×10^{-4}	20	0.80×10^{3}	
17.5×10^{-4}	20	0.686×10^{3}	
20×10^{-4}	20	0.60×10^{3}	
10×10^{-4}	10	0.6×10^{3}	4.198×10^{3}
11.11×10^{-4}	10	0.54×10^{3}	
12.22×10^{-4}	10	0.49×10^{3}	
13.33×10^{-4}	10	0.45×10^{3}	
14.44×10^{-4}	10	0.416×10^{3}	
15.55×10^{-4}	10	0.386×10^{3}	
16.66×10^{-4}	10	0.360×10^{3}	
17.77×10^{-4}	10	0.338×10^{3}	
18.88×10^{-4}	10	0.318×10^{3}	
20×10^{-4}	10	0.30×10^{3}	

EFFECT OF NON-NEWTONIAN COOLING.

The Newtonian cooling of solids presupposes that the resistance to heat transfer through conduction inside the body is negligible. When the heat flows from the interior of such a body to a surrounding fluid, the entire resistance to heat flow is offered by the fluid film. For the purpose of finding out how far the assumption of Newtonian cooling in the case under study is tenable, let us consider a heat generating sphere of radius 'a'. Let k represent the thermal conductivity of the material, q, the rate of internal heat generation, T_s, the surface temperature and T, the inside temperature of the sphere at any point.

Let us consider a very small shell of thickness dr, inside the sphere.

In the steady state, the sum of the heat received by the shell from the interior plus that generated within the shell itself must be equal to the heat flowing out of the shell.



$$-4\pi k r^2 \frac{dT}{dr} + 4\pi q r^2 dr = -4\pi k \left[r^2 \frac{dT}{dr} + \frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) dr \right]$$

$$\therefore \frac{q r^2 dr}{k} + d\left(r^2 \frac{d-}{dr}\right) = 0$$

Integrating,

$$\frac{q r^3}{3k} + r^2 \frac{dT}{dr} = C_1 .$$

$$dT = C_1 \frac{dr}{r^2} - \frac{q\,r\,dr}{3K}$$

$$\therefore T = -\frac{C_1}{r} - \frac{q r^2}{6K} + C_2 .$$

Boundary Conditions:

T is finite when $r = 0$

$\therefore C_1 = 0$

$T = T_s$ when, $r = a$

$\therefore T_s = C_2 - \frac{q a^2}{6K}.$

$\therefore C_2 = T_s + \frac{q a^2}{6K}.$

$$\therefore T = T_s + \frac{q a^2}{6K} - \frac{q r^2}{6K}.$$

$$\therefore T = T_s + \frac{q}{6K}(a^2 - r^2) \quad . \; . \; . \; . \; . \; . \; . \; . \; (11)$$

Temperature at the center $= T_s + \frac{q a^2}{6K} \; . \; . \; . \; . \; (12)$

Example problems:

$k = 25$ Btu/hr. °F.Ft^2/Ft.

$T_s = 1000$°F

(a) $q = 100{,}000$ Btu/hr.Ft^3

$a = .01$ Ft.

Temperature at the center $= 1000 + \frac{100{,}000 \times .01^2}{6 \times 25} = 1000.06$°F

(b) $q = 1{,}000{,}000$ Btu/hr.Ft^3

$a = .01$ Ft.

Temperature at the center $= 1000 + \frac{1{,}000{,}000 \times .01^2}{6 \times 25} = 1000.6$°F (c), $q = 100{,}000$ Btu/hr.Ft^3

$a = .01$ Ft.

Temperature at the center $= 1000 + \frac{100{,}000 \times .01^2}{6 \times 25} = 1006.6°F$

(d) $q = 1{,}000$ Btu/hr.Ft^3

$a = .01$ Ft.

Temperature at the center $= 1000 + \frac{1{,}000{,}000 \times .01^2}{6 \times 25} = 1066.6°F$

(e) $q = 1.5 \times 10^8$ Btu/hr.Ft^3

$a = .01$ Ft.

Temperature at the center $= 1000 + \frac{1.5 \times 10^8 \times 101^2}{6 \times 25} = 1100°F$

(f) $q = 1.5 \times 10^8$ Btu/hr.Ft^3

$a = .01$ Ft.

Temperature at the center $= 1000 + \frac{1.5 \times 10^8 \times .01^2}{6 \times 25} = 11000°F$

The above calculations clearly show that in the case of very small spheres, the error introduced by assuming Newtonian cooling is not considerable. This is especially true when the rate of heat generation is not excessively large. In the extreme cases where the grain size is not too small and the rate of internal heat generation is considerably high, the assumption of Newtonian cooling should be made only with caution.

EFFECT OF THE DIRECTION OF COOLANT FLOW ON THE MAXIMUM TEMPERATURE OF THE COOLANT.

In general, in heat transfer processes the coolant may be passed parallel to the direction of heat flow or in the opposite direction. Cross flow systems also are in practice. In the case of transpiration cooling of a porous plate, the direction of coolant flow has no effect on the maximum temperature of the coolant.

V. CONCLUSIONS

This investigation of the transpiration cooling of a heat generating porous plate has been based on more realistic assumptions as compared to previous studies. The analysis has been more rigorous. By virtue of these, the results are more reliable.

The analysis of the effect of the specific heat of the coolant has conclusively proved the desirability of choosing coolants of high specific heat. An interesting observation that this investigation has brought out is the conflicting effects of the grain size on the heat transfer surface area and on the temperature gradient. It may be recalled here that as the grain size increases the temperature gradient increases; but, only at the expense of the heat transfer surface area. This necessitates striking of an optimum grain size so as to make the best out of this paradoxical situation. This intriguing phenomenon might be worth further considerations.

The observations regarding the direct and indirect influence of the compressibility of the coolants-viz. the advantages of high pressure systems from the point of view of pressure drop, pumping power and heat transfer rate must be of significance. Here again, investigations regarding optimum conditions might prove worthwhile.

The investigation has succeeded in proving that in the case of small spheres, the assumption of Newtonian cooling does not jeopardize the accuracy of the results.

Further investigation into cases where the porous plate is composed of grains other than spherical in shape is suggested. Study of heat transfer through porous bodies other than plates seems to be another useful project. Experimental verification of the analytical results obtained so far appears to be inviting.

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VII. APPENDIX

TABLE A. 1

SPECIFIC HEAT OF COOLANT FLUIDS (24,25)

Fluid	Temperature °F.	Specific Heat Btu/lb. °F.
Helium (1 atm)	all temp.	1.240
Dry air (1 atm)	45	.24
	243	.242
	441	.246
	639	.252
	1250	.270
	2060	.287
Carbondioxide	212	.22
	570	.23
	750	.28
Nitrogen	32	.25
	2550	.31

APPENDIX

TABLE A. 2

THERMAL CONDUCTIVITIES OF SOME SOLID MATERIALS USED IN NUCLEAR REACTORS (24)

Material	Thermal Conductivity K Btu/hr.°F.Ft^2/Ft.	Temp. °F
Aluminum (25)	128	77
Beryllium (extruded)	80	200
	73	400
	68	600
	61	800
Berylliumoxide	29	400
	22	600
	18	800
Graphite	76	600
	65	800
	59	1000
Stainless Steel (type 347)	9	212
	12.6	932
Thorium	22	212
	26	1202
Uranium (cast)	15	200
	17.5	400
	18	600
Zirconium (crystal bar)	11.8	212
	11.4	392

APPENDIX

TABLE A. 3

SURFACE AREA PER UNIT VOLUME FOR DIFFERENT GEOMETRICAL SHAPES (9)

Shape	Area Per Unit Volume	Remarks
Sphere	$\frac{6}{d}$	d = diameter
Cube	$\frac{6}{d}$	d = length of edge
Oblate spheroid	$\frac{7.64}{d}$	Thickness = $\frac{\text{diam.}}{2}$
Prolate spheroid	$\frac{5.18}{d}$	length = 2 x diam.
	$\frac{4.94}{d}$	length = 3 x diam.
	$\frac{4.84}{d}$	length = 4 x diam.

APPENDIX

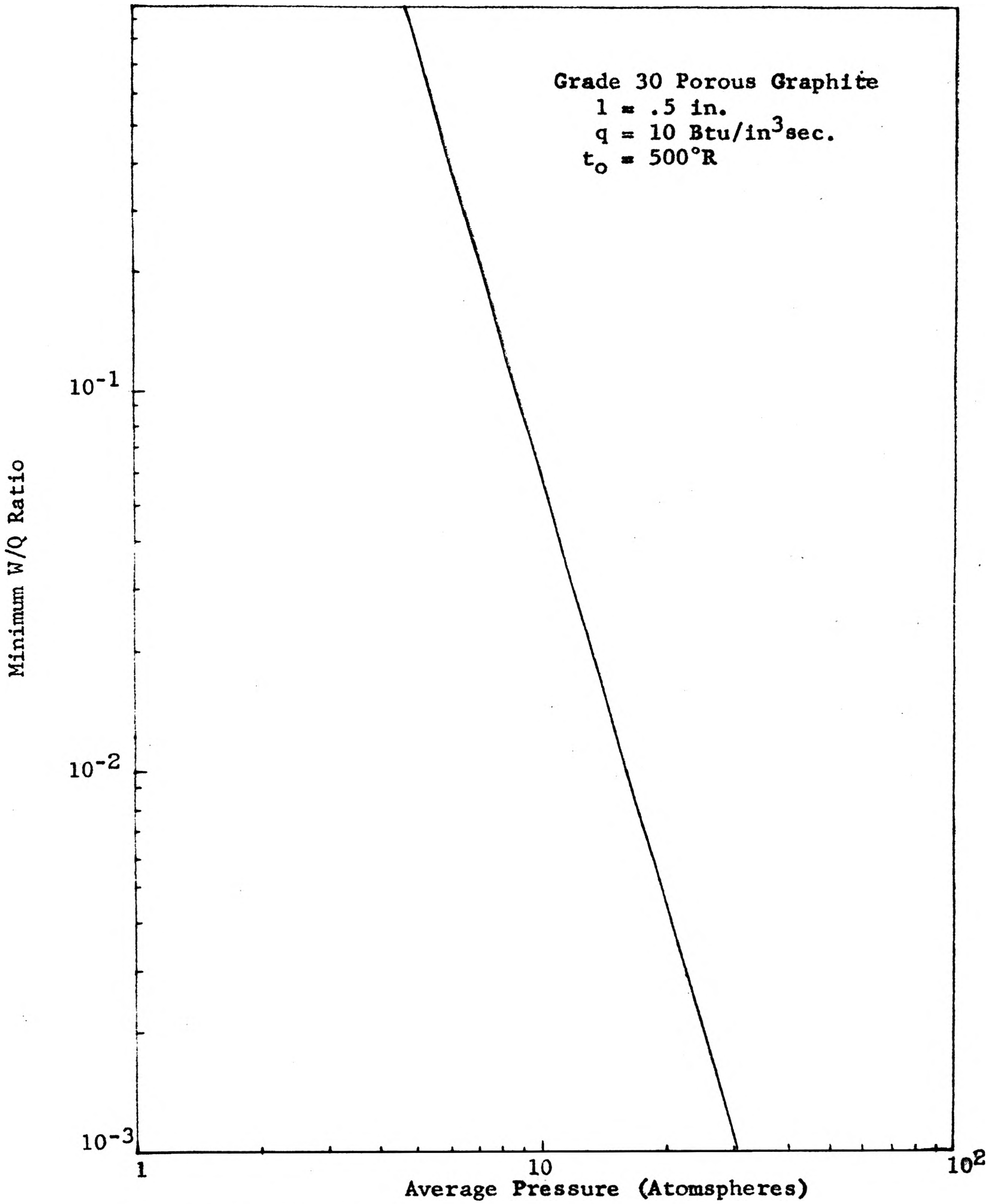

Fig. A. 1 Minimum W/Q Ratio Vs Average Pressure (After Leon Green, Jr. 3.)

APPENDIX

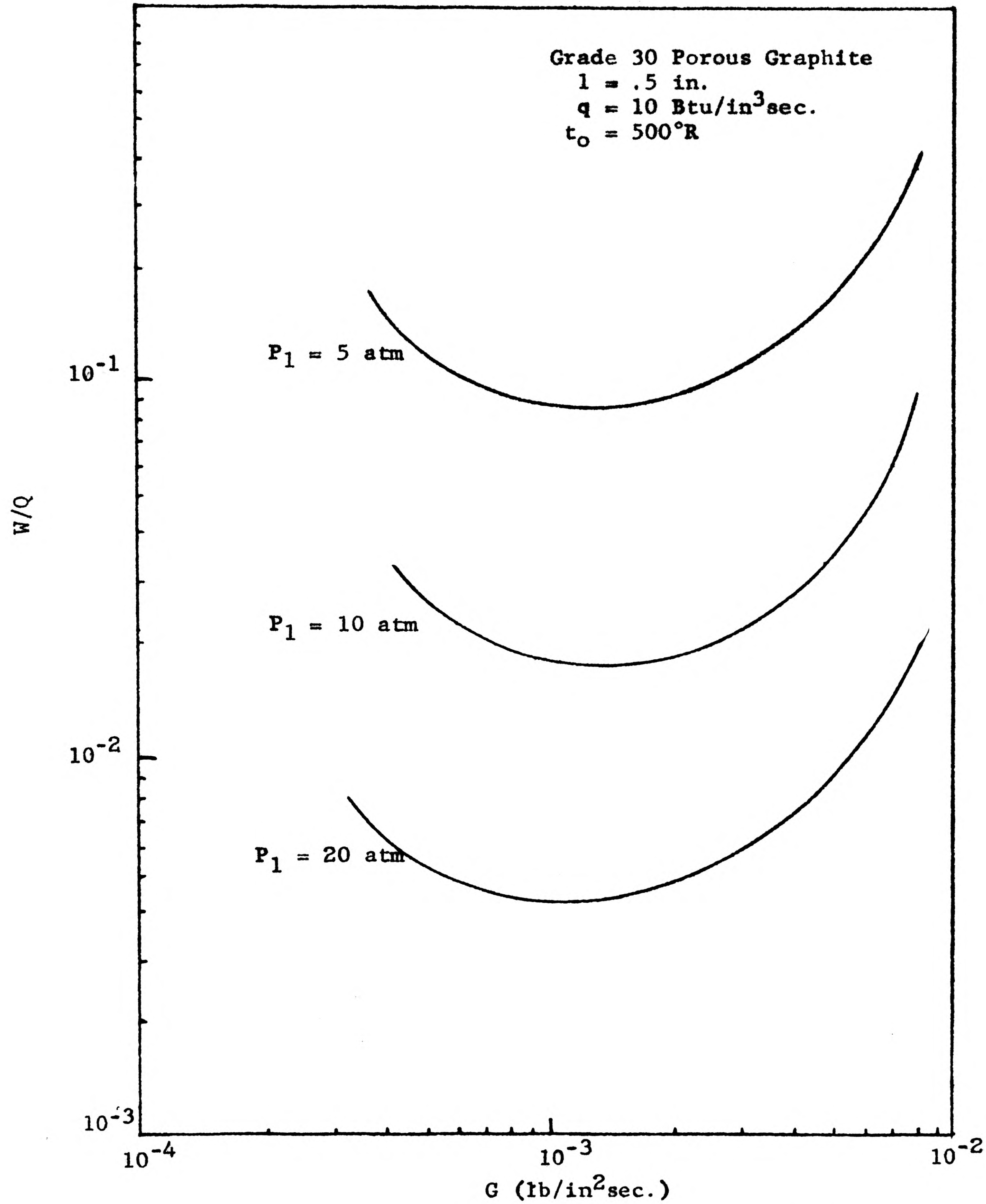

Fig. A. 2 Pumping-Power/Power-Output Ratio Vs Flow Rate For Different Inlet Pressure (After Leon Green, Jr. 3.)

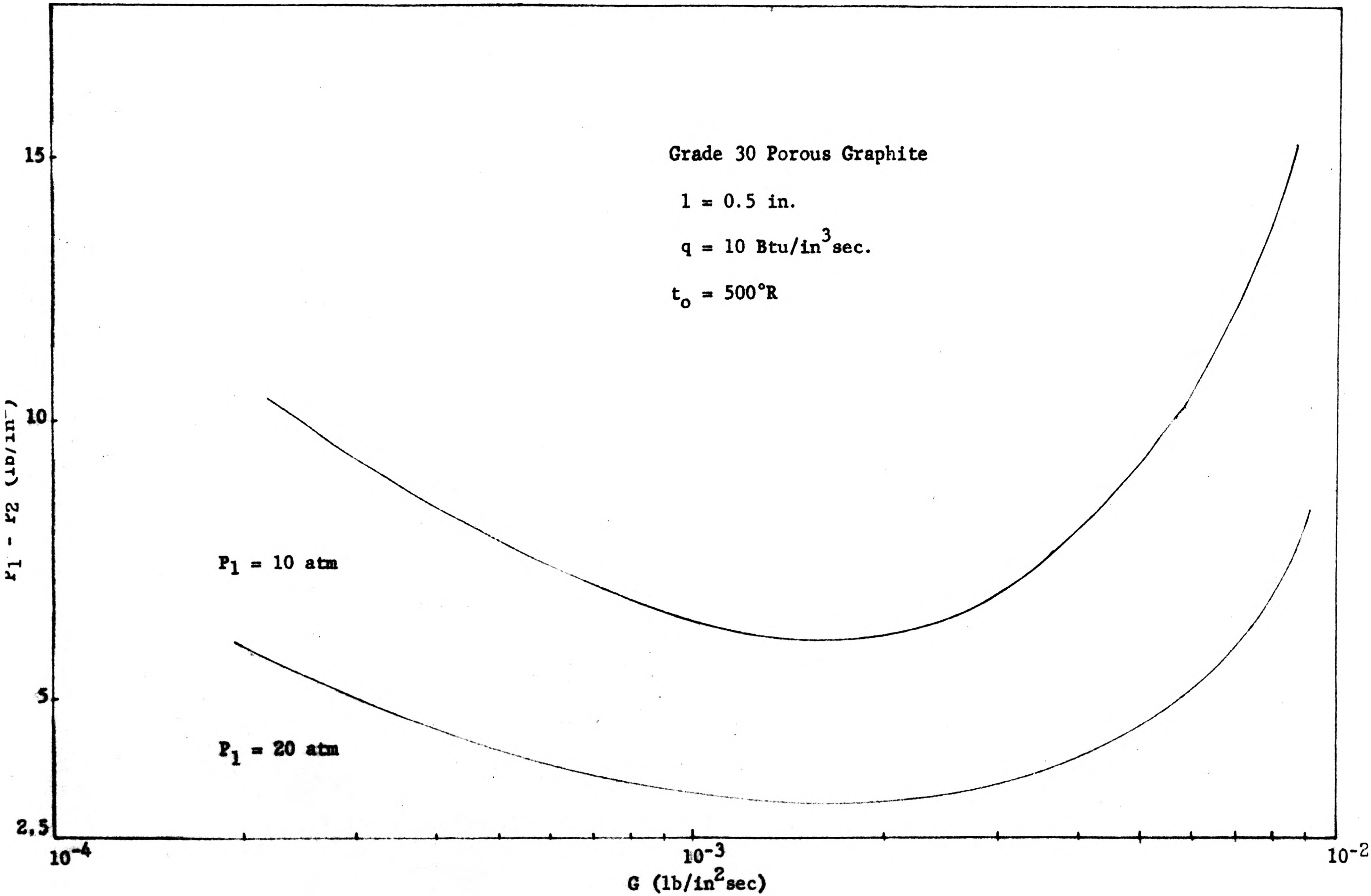

Fig. A. 3 Pressure Drop Across Wall Vs Flow Rate (After Leon Green, Jr. 3.)

APPENDIX

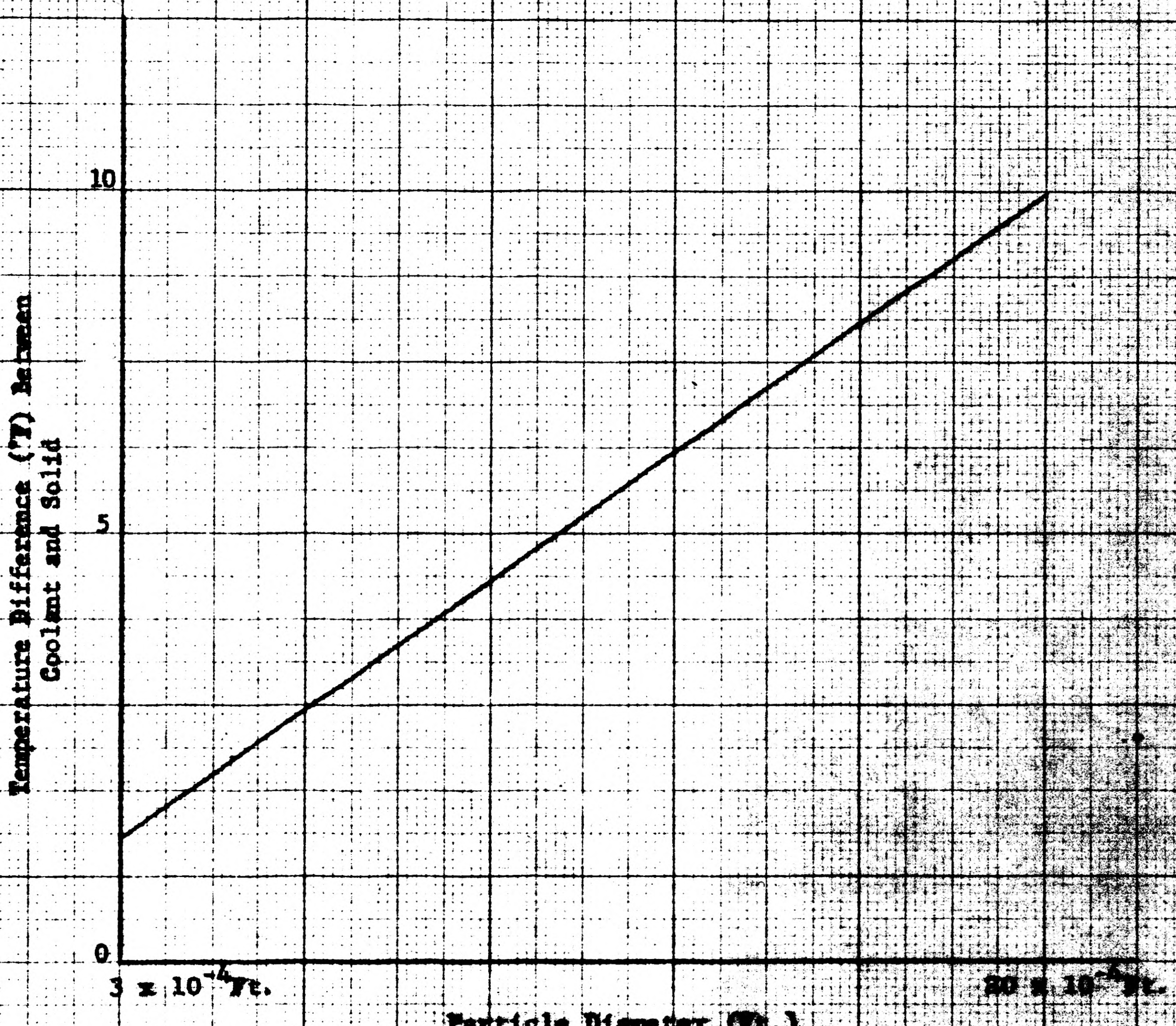

Fig. A.4 Temperature Difference Between The Solid And The Coolant Through The Porous Plate With Uniform Distribution of Heat Generation. (After T. S. Furumal. 7.)

VIII. VITA

The author was born on first of October 1930. He had his elementary, middle school and high school education in his native town of Alwaye, India. He continued his education at Collegiate level under the Madras University and claimed Bachelor of Science degree in 1951 and the degree of Bachelor of Science in Technology in 1954. This was followed by a short period of practical training at the plants of the Fertilisers and Chemicals Travancore Ltd. He later served the same company as Design Engineer and subsequently assumed additional responsibility as project engineer in charge of the Company's expansion program. From August 1958 he is attached to the Thangal Kunju Musaliar College of Engineering at Quilon, India, as the Vice-Principal.

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